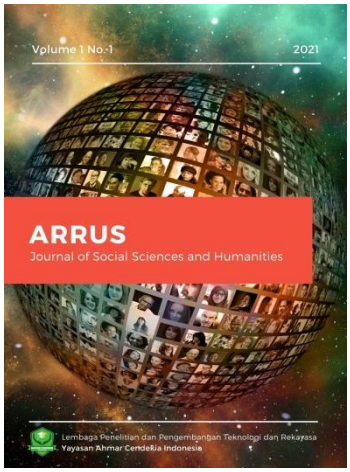
https://doi.org/10.35877/soshum5142

OPEN ACCESS

ISSN: 2776-7930 (Print) / 2807-3010 (Online)



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RESEARCH ARTICLE

An Interactive and Dynamic Algebra Learning Environment: A Six-Principle Design Framework Validated Through Design-Based Research

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Abstract: University-level Linear Algebra is widely recognised as the most challenging transition point in undergraduate mathematics, yet technology-enhanced learning environments for this course remain under-researched. This paper presents the Interactive and Dynamic Algebra Learning Environment (IDALE), a six-principle design framework developed and empirically validated through a full Design-Based Research (DBR) cycle at Universitas Negeri Makassar, Indonesia, during the 2023/2024 academic year. The study involved 36 students in an experimental class and 26 in an independent replication class across 14 instructional sessions. IDALE integrates six interdependent principles, each grounded in an established learning theory: Dual Representation Integration, Digital System-Based Self-Exploration, Contextual and Task-Based Collaboration, Integrated Axiomatic-Procedural Learning, Instant Feedback and Structured Validation, and Adaptive Dynamics and Learning Autonomy. These principles are enacted through a five-phase learning trajectory spanning mindful, joyful, and meaningful deep learning dimensions. Empirical results demonstrated a 97.2% pass rate against a 65% benchmark, 97.2% cumulative positive student response, and consistent active engagement of at least two-thirds of students from Session 5 through Session 14. These gains are reported descriptively: the study's design-based, single-institution scope is treated as an explicit boundary condition on generalisability, and confirmatory inferential statistical analysis is identified as a priority for future validation cycles. Replication class findings corroborated cross-instructor transferability at a qualitative, descriptive level. IDALE constitutes a theoretically grounded, practically actionable, and empirically validated framework for designing interactive digital mathematics learning environments in higher education.

Keywords: IDALE, Learning Environment, Algebra

1. Introduction

University-level Linear Algebra occupies a structurally pivotal position in undergraduate mathematics: it constitutes the primary gateway through which students transition from computational, procedural thinking to abstract, axiomatic reasoning (Sierpinska, 2000; Dorier et al., 2000). This transition is notoriously demanding. Students who have succeeded in procedural calculus frequently encounter a profound cognitive discontinuity when confronted with vector spaces, eigenvalue theory, and linear transformations - topics demanding what Tall (2008) terms 'formal-axiomatic' thinking, a qualitatively different cognitive register from the operational reasoning that earlier courses reward.

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https://doi.org/10.35877/soshum5142

ISSN: 2776-7930 (Print) / 2807-3010 (Online)

Despite decades of research documenting these difficulties (Kuru, 2022; Göller et al., 2024), the instructional environment in which Linear Algebra is taught has changed relatively little in most institutions. Predominantly lecture-based instruction supplemented by worked examples systematically under-supports the multi-representational, exploratory, and reflective processes that genuine algebraic understanding requires. The physical and digital environment in which learning takes place - as a deliberately designed object - receives insufficient scholarly and practical attention.

Accessible digital tools - collaborative spreadsheets, dynamic geometry software, and virtual meeting platforms - open new possibilities for designing algebra learning environments that are simultaneously interactive and dynamic. Yet few studies have operationalised these twin properties into a coherent, replicable, and empirically validated design framework. Effective technology integration in mathematics teaching depends on aligning tool choice with pedagogical practice rather than technology alone (Rakes et al., 2022). General exhortations to 'use technology' or 'promote collaboration' remain too vague to guide instructional design; practitioners need specific, theoretically grounded principles specifying how digital tools, collaborative structures, instructional sequencing, and feedback practices should be organised. This question of tool choice is unresolved in the literature: specialised dynamic mathematics platforms such as GeoGebra, MATLAB, and Maple offer purpose-built symbolic and graphical affordances, yet carry real adoption barriers - installation, licensing, and a steeper learning curve - that general-purpose, already-familiar collaborative tools do not (Solano-Macías, 2023; Ishartono et al., 2022; Engelbrecht & Borba, 2024). Whether the pedagogical value of a design framework such as IDALE derives from its six design principles or from the specific software used to implement them has not been directly examined.

This paper addresses that gap by presenting the Interactive and Dynamic Algebra Learning Environment (IDALE), a six-principle instructional design framework developed and validated through a full Design-Based Research (DBR) cycle. Four research questions guided the study, the fourth added specifically to address the choice of general-purpose collaborative tools over specialised dynamic mathematics software:

(1). RQ1: How can an interactive and dynamic design framework for university algebra learning be developed through DBR?
(2). RQ2: What are the defining principles and environmental characteristics of the resulting IDALE framework?
(3). RQ3: How effective is IDALE in supporting learning outcomes, student responses, and active participation?
(4). RQ4: What unique cognitive and collaborative affordances do general-purpose spreadsheet and video-conferencing tools (Google Sheets, Google Meet) offer for the IDALE design principles, relative to specialised dynamic mathematics platforms such as GeoGebra, MATLAB, or Maple?

The paper makes three original contributions: (a) a six-principle design framework grounded in six established learning theories; (b) a five-phase empirically derived learning trajectory for Linear Algebra; and (c) convergent validity evidence from an experimental class and an independent replication class.

2. Literature Review

2.1. The Problem of Learning Linear Algebra

Dorier et al. (2000) locate the core difficulty of Linear Algebra in its 'unifying and generalising' epistemological role: vector spaces subsume prior mathematical objects within a single abstract structure, yet students typically lack experience with axiomatic generalisation. Sierpinska (2000) identifies three non-equivalent cognitive modes required: synthetic-geometric, analytic-arithmetic, and analytic-structural. Most students arrive fluent in only the

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https://doi.org/10.35877/soshum5142

ISSN: 2776-7930 (Print) / 2807-3010 (Online)

first two; the analytic-structural mode - treating algebraic structures as objects with properties rather than procedures to execute - must be cultivated by deliberate instructional design.

Technology-enhanced environments show significant promise in mediating this transition. Dynamic software enables students to visualise eigenvector transformations and vector space geometry in real time (Orhani, 2024; Çağlayan, 2015). Spreadsheet-based parameter manipulation allows students to observe relationships between structural choices and computational outcomes, bridging procedural and formal understanding (Urhan & Zengin, 2024). Explicit multi-representational instruction improves both performance and self-efficacy in Linear Algebra (Acikyildiz & Kosa, 2021b); comparable technology-mediated linear algebra teaching has been shown to sustain learner-centred interaction even in fully online delivery (Kontorovich, 2024), and targeted instructional models have measurably improved linear algebra performance among prospective mathematics teachers in the Indonesian context (Tristanti & Nusantara, 2023).

2.2. *Interactive and Dynamic Learning Environments*

Moore (2001) identifies three interaction types: learner-content, learner-instructor, and learner-learner. Uppal and Ali (2022) extend this into three interactivity dimensions: system interactivity (active learner participation within the technological system), service interactivity (instruction as facilitation rather than transmission), and content interactivity (materials that connect to and extend prior knowledge). An interactive environment activates all three dimensions in concert, consistent with broader principles for designing constructivist interactive learning environments (Davies & Wilson, 2020). Dynamism, as defined by Cevikbas and Kaiser (2021), refers to an environment that initiates constant change in learners' knowledge, capability, and proficiency through varied formats, adaptive sequencing, continuous feedback, and self-paced provisions (Gambo & Shakir, 2021).

2.3. *Six Theoretical Pillars of IDALE*

Each IDALE principle is grounded in an established learning theory:

(a). Dual Coding Theory (Paivio, 1986) - information encoded through both verbal and non-verbal channels is more deeply processed; IDALE operationalises this through systematic multi-representational task design.

(b). Digital Constructivism (Jonassen, 1995) - digital technologies should function as 'mindtools' extending learners' thinking capacity; IDALE implements this through spreadsheet-based self-exploration.

(c). Social Constructivism (Vygotsky, 1978; Palincsar, 1998; Bringula & Atienza, 2023) - conceptual development is mediated through social interaction; IDALE operationalises this through structured collaborative tasks and peer validation.

(d). Structural Theory in Mathematics (Tall, 2008) - advanced mathematics requires integration of procedural and formal-axiomatic understanding; IDALE addresses this through axiomatic-procedural integrated sequences.

(e). Formative Assessment Theory (Black & Wiliam, 2009; Barana et al., 2021; Palm, 2023) - immediate, specific feedback accelerates metacognitive development; IDALE implements this through automated spreadsheet validation and peer-review protocols.

(f). Self-Regulated Learning Theory (Zimmerman, 2002; Göller et al., 2024; Bednorz & Bruhn, 2023) - learning autonomy is a prerequisite for effective self-directed learning; IDALE fosters this through adaptive pacing and multi-modality task access.

3. Research Method and Materials

3.1. *Design-Based Research*

DBR was selected because the research goal was to co-construct, through empirical cycles, both the IDALE framework and the local instructional theory accounting for its workings

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(Gravemeijer & Cobb, 2006; Leonard, 2023; Tinoca et al., 2022). DBR is characterised as interventionist, iterative, process-oriented, utility-oriented, and theory-oriented. Four stages structured the study: (1) Preparation - literature analysis, learner diagnosis, and initial HLT construction; (2) Thought Experiment - expert validation of the HLT; (3) Teaching Experiment - classroom enactment across 14 sessions with iterative micro-design revision; (4) Retrospective Analysis - systematic cross-time, cross-case matrix analysis to derive IDALE principles.

3.2. *Setting and Participants*

The study was conducted in the 2023/2024 academic year at the Department of Mathematics, Universitas Negeri Makassar (UNM), Indonesia, within the compulsory Elementary Linear Algebra course. An experimental class (n = 36, third-year pre-service mathematics teachers) participated in the full DBR cycle. A replication class (n = 26), taught by a different instructor using the finalised IDALE design, provided cross-instructor validity evidence. Four specialist lecturers served as expert reviewers during the thought experiment.

3.3. *Instruments*

Table 1. Research instruments, functions, and validation

Instrument	Function	Validation
Hypothetical Learning Trajectory (HLT)	14-session blueprint of goals, activities, and expected learning processes	Two-cycle expert review; CVR ≥ 0.75
Interview Guidelines	Student and instructor perspectives on learning quality	Expert panel content validation
Field Notes (Observation)	Systematic recording of interactivity dimensions per session	Observer protocol inter-rater agreement
Learning Outcome Tests (UTS & UAS)	Mid- and end-semester achievement assessment	Expert review; content alignment with HLT
OTL Response Questionnaire (16 items)	Perceived quality of Opportunity to Learn provisions	CVR ≥ 0.75; 4-point Likert scale
IDALE Response Questionnaire (13 items)	Perceived quality of the interactive learning environment	CVR ≥ 0.75; 4-point Likert scale
Activity Observation Sheet	Proportion and quality of active student engagement per session	Observer reliability protocol
Student Worksheets	In-class and digital (spreadsheet) task records	Content alignment with HLT topics

Instrument validation in this DBR cycle relied on expert-panel content validity (CVR ≥ 0.75) rather than large-sample psychometric scaling, since both the 16-item OTL and 13-item IDALE questionnaires were purpose-built formative instruments administered within a single semester-length cohort. Internal consistency coefficients (e.g., Cronbach's alpha) were therefore not computed for this cycle; this is noted as an explicit limitation in Section 5.3, particularly given ongoing debate about when alpha is an appropriate and sufficiently informative reliability index for short, purpose-specific classroom instruments (Zakariya, 2022; Dorsah, 2026). Future administrations of the OTL and IDALE questionnaires across larger, pooled cohorts will compute alpha (or a more appropriate reliability index) alongside the CVR-based content validation reported here, following recommended practice for combining content- and reliability-based evidence in instrument validation (Masuwai et al., 2024).

3.4. *Instructional Design and Digital Integration*

Topics were covered across 14 sessions: vector spaces and subspaces (S1-S3), linear combinations, basis and dimension (S4-S6), rank and nullity (S7), mid-semester assessment (S8), inner products (S9-S10), eigenvalues and eigenvectors (S11-S12), linear transformations (S13-S14). Digital integration was implemented progressively: Google Sheets templates for vector space axiom verification were introduced from S3; virtual collaborative exploration via Google Meet combined with shared spreadsheet workspaces was introduced in S9-S14.

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ISSN: 2776-7930 (Print) / 2807-3010 (Online)

Face-to-face and virtual modes operated as complementary rather than substitutive environments.

3.5. *Data Analysis*

Quantitative data were analysed using descriptive statistics. Student response data were evaluated against a cumulative positive threshold of ≥ 75%. Activity engagement was categorised per session into three bands: <1/3, 1/3-2/3, and ≥2/3 of students actively engaged. Qualitative data - field notes, interviews, worksheets - underwent four-stage thematic analysis: open coding, categorisation, reflective interpretation, and triangulation across sources and time points, following the matrix analysis procedure of Gravemeijer and Cobb (2006). Trustworthiness was established through source triangulation, time triangulation, member checking, and ecological validity (authentic unmodified course setting). This combination of aptitude-based outcome measures with process-level engagement observation follows recent recommendations for triangulating self-regulated learning evidence in digital mathematics classrooms (Zhidkikh et al., 2023).

3.6. *The IDALE Framework*

3.6.1. *Overview*

IDALE emerged from retrospective analysis of the full DBR cycle as a design framework comprising six interconnected principles, eight realised environmental characteristics, and a five-phase learning trajectory. The framework is organised around two conceptual axes: an interactivity axis (quality of learner engagement with content, peers, and the system) and a dynamism axis (adaptive, temporally evolving nature of the environment). Principles operate at the design level; characteristics describe the realised environment; and the five-phase trajectory provides temporal sequencing architecture.

3.6.2. *Six Principles of IDALE*

Table 2. Six **IDALE** principles: definitions, theoretical bases, and classroom instantiations

#	Principle	Core Definition	Theoretical Basis	Classroom Instantiation
P1	Dual Representation Integration	Algebraic and visual/digital representations are systematically linked across every content unit	Paivio (1986); Goldin (2002)	Students connect symbolic matrix equations to spreadsheet-generated numerical and visual outputs; simultaneous algebraic and geometric treatment of all topics
P2	Digital System-Based Self-Exploration	Digital tools function as cognitive scaffolds enabling autonomous exploration and hypothesis testing	Jonassen (1995)	Students adjust spreadsheet parameters to observe changes in system solutions, determinants, eigenspaces; automated cell validation enables self-correction without instructor mediation
P3	Contextual and Task-Based Collaboration	Collaborative activities are anchored in challenging, reflective tasks in face-to-face and virtual settings	Vygotsky (1978); Palincsar (1998)	Structured group tasks via Google Meet and shared Sheets; written justification requirements for divergent answers; joint portfolio construction
P4	Integrated Axiomatic-Procedural Learning	Procedural execution and axiomatic verification are taught as inseparable, mutually reinforcing mathematical activities	Tall (2008)	Axiom verification follows computational spreadsheet exploration rather than preceding it; proof-writing is motivated by observed computational patterns
P5	Instant Feedback and Structured Validation	The environment provides immediate, targeted feedback accelerating metacognitive regulation	Black & Wiliam (2009); Barana et al. (2021)	Conditional cell formatting in Google Sheets flags correct/incorrect entries in real time; structured peer-review protocols on written proofs
P6	Adaptive Dynamics and Learning Autonomy	Students control pace, strategy selection, and media choice within a structured curriculum	Zimmerman (2002)	Multi-device task access; choice of paper or digital modality; self-paced exploration blocks within session time allocations

3.6.3. *Eight Characteristics of the IDALE Environment*

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The six principles are realised in a learning environment exhibiting eight observable characteristics:

(a). Freedom of Self-Exploration: students freely investigate algebra concepts at their own pace using digital tools, building intrinsic motivation and intellectual confidence.
(b). Personal Learning Strategies: the environment accommodates symbolic, visual, and numerical reasoning pathways, respecting individual cognitive style.
(c). Self-Paced Rhythm: no rigid within-session pacing constraints; students regulate progress through task sequences.
(d). Adaptive Dynamics: learning activities and instructor interventions adjust in real time to observed student progress and emerging misconceptions.
(e). Learning Autonomy: students exercise deliberate responsibility for goal-setting, strategy selection, and self-monitoring.
(f). High-Level Cognitive Activity: tasks consistently require analysis, synthesis, evaluation, and generalisation rather than reproductive execution.
(g). Collaborative Reflection and Validation: structured peer discussion, cross-checking, and joint argumentation are embedded as routine instructional activities.
(h). Digital Technology as Scaffold: spreadsheets and virtual platforms function as cognitive partners that extend mathematical reasoning rather than merely deliver content.

3.6.4. Five-Phase Learning Trajectory

Table 3. Five-phase IDALE learning trajectory

Phase	Sessions	Deep Learning Dimension	Primary Activities	Learning Goals
Phase 1: Procedural Foundation	S1-S2	Mindful	Diagnostic assessment; structured problem-solving; reflective why/how prompts; axiom identification tasks	Establish metacognitive awareness and self-regulation as foundations for subsequent learning
Phase 2: Digital Exploration	S3-S5	Joyful + Meaningful	Spreadsheet-based vector space exploration; individualised digital tasks; creative problem reformulation	Foster intrinsic motivation, autonomous exploration, and connection of algebra to real contexts
Phase 3: Collaborative Diversification	S6-S7	Joyful + Mindful + Meaningful	Group problem-solving; peer argumentation; proof construction; written justification tasks; peer validation protocols	Develop critical reasoning and collaborative mathematical discourse; equalise engagement
Phase 4: Peak Interactivity	S9-S12	Meaningful + Joyful + Mindful	Project-based tasks; Google Meet collaborative Sheets; eigenvalue/transformation exploration; peer portfolio validation	Synthesise conceptual and procedural knowledge; develop transfer capacity and creative problem-solving
Phase 5: Reflection & Generalisation	S13-S14	Mindful + Meaningful	Concept mapping; cross-domain application; portfolio compilation; structured self-assessment	Consolidate generalised mathematical understanding; develop foundational self-regulation dispositions that support continued mathematical learning beyond the course

4. Results and Discussion

4.1. Results

4.1.1. Development of IDALE Through DBR Stages

(1). Preparation

Preliminary analysis of 2022/2023 cohort performance revealed a bimodal grade distribution with a persistent low-achieving cluster. Diagnostic interviews attributed this to passive instruction and insufficient scaffolding for abstract concepts. Task analysis across 13 content

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https://doi.org/10.35877/soshum5142

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topics structured 14 instructional sessions. The initial HLT embedded Google Sheets exploration tasks from S3 and introduced virtual collaborative sessions from S9.

(2). Thought Experiment

Expert review across two revision cycles generated three major revisions: (a) extending S3-S4 time to accommodate axiom exploration; (b) embedding explicit reflective prompts in every session; (c) adding written peer-validation sub-tasks in S6, S9, and S12. All final HLT validation scores exceeded acceptability thresholds.

(3). Teaching Experiment - Key Iterative Revisions

(a). S2-S3 Revision: Students focused on spreadsheet navigation rather than algebraic reasoning. Revision added mandatory paper-based axiom listing before digital tasks, implementing P4 more forcefully and ensuring conceptual grounding precedes digital exploration.

(b). S5-S6 Revision: Collaboration was surface-level - answers shared without argumentative discourse. Revision restructured tasks to require written justification of divergent group answers, operationalising P3 and generating genuine sociomathematical dialogue (Yackel & Cobb, 1996).

(c). S11-S12 Revision: Students computed eigenvalues but could not connect them to geometric transformations. Revision introduced dynamic spreadsheet templates visualising eigenvector directions under transformation, activating P1 and P2 to bridge procedural and structural understanding.

(4). Retrospective Analysis

Cross-session matrix analysis of all 14 sessions yielded the six IDALE principles as the recurring design features consistently distinguishing high-quality engagement sessions from lower-quality ones. The five-phase trajectory emerged from temporal analysis: engagement proportions and learning quality indicators showed a clear developmental arc from passive individual work (Phase 1) to sustained collaborative deep learning (Phase 5). These structural patterns were consistent across both the experimental and replication classes.

4.1.2. Effectiveness Evidence

(1). Learning Outcomes

Table 4. Learning outcome statistics - experimental class (n = 36)

Parameter	UTS (Mid-semester)	UAS (End-semester)	Change
Class mean	84.7	89.5	+4.8
Highest score	98	100	+2
Lowest score	40	52.5	+12.5
Standard deviation	13.8	11.5	-2.3
Grade A / A+ (n)	11	19	+8
Grade C / C+ (n)	6	3	-3
Grade D (n)	1	1	0
Pass rate	97.2%	97.2%	--

The 97.2% pass rate substantially exceeded the 65% effectiveness benchmark. The +4.8-point mean increase, the growth from 11 to 19 Grade A/A+ students, and the standard deviation reduction from 13.8 to 11.5 collectively indicate improvement and equalisation of achievement - a signature effect of IDALE's differentiation provisions (P6) enabling lower-achieving students to accelerate more than high-achieving peers. These comparisons are reported descriptively rather than inferentially: the dataset available for this cycle did not support paired significance testing (e.g., a paired-samples t-test between UTS and UAS scores) or standardised effect-size estimation (e.g., Cohen's d), and the reported gain should

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ISSN: 2776-7930 (Print) / 2807-3010 (Online)

accordingly be read as a directional, practically meaningful trend rather than a statistically confirmed effect. This is addressed as a limitation in Section 5.3.

(2). Student Responses to IDALE

Table 5. Frequency distribution of student responses to IDALE questionnaire (n = 36)

Response Category	n	%	Min. Criterion
Positive (mean ≥ 3.5/4.0)	15	41.7%	
Trending Positive (3.0-3.49)	20	55.6%	
Trending Negative (2.5-2.99)	1	2.8%	
Negative (< 2.5)	0	0.0%	
Cumulative Positive	35	97.2%	≥ 75%

The 97.2% cumulative positive response substantially exceeded the 75% minimum. Of 36 students, 25 (69.4%) provided universally positive responses across all 13 questionnaire items. The single trending-negative response was traced in follow-up interview to limited prior digital experience - an individual factor rather than a systemic design failure, but one signalling the importance of digital literacy prerequisites in IDALE implementation.

(3). Student Activity Engagement

Table 6. Session-by-session student engagement levels

Session(s)	Engagement Level	Dominant Interaction Mode
S1	< 1/3	Individual, instructor-directed; minimal peer interaction
S2-S4	1/3-2/3	Emerging small-group discussion; partial spreadsheet use; uneven participation
S5-S6	≥ 2/3	Reflective group discourse; broad digital engagement; peer validation emerging
S7	1/3-2/3	Active but not yet fully distributed across class
S9-S14	≥ 2/3 (consistent)	Sustained collaborative exploration; argumentative discourse; joint digital portfolio work; no decline

From S5 onward, active engagement of ≥ 2/3 of students was achieved in every session except S7, with full consistency from S9 through S14. Critically, no engagement decline was observed across the second half of the semester, confirming that IDALE built a self-sustaining participatory culture rather than producing transient initial enthusiasm. The engagement arc mirrors the five-phase IDALE structure, with the most intensive collaborative activity beginning in Phase 3 and reaching its peak in Phase 4.

(4). Replication Class

The replication class (n = 26, independent instructor) showed qualitatively comparable engagement trajectories and learning progression patterns: an early passive phase transitioning to active collaboration from the fourth session; consistent use of spreadsheets as a reasoning rather than procedural tool; and student-initiated voluntary elaboration of formal tasks - a hallmark of deep learning (Biggs, 1999). Minor pacing variations reflecting the instructor's stylistic preferences did not alter the fundamental learning trajectory, supporting IDALE's cross-instructor transferability - a pattern consistent with other recent inquiry-based, technology-mediated redesigns of linear algebra instruction (Fredriksen et al., 2024). This comparison is qualitative and descriptive rather than inferential: no formal between-class significance test (e.g., an independent-samples t-test or Mann-Whitney U test) was conducted on UTS/UAS or questionnaire scores, so the claim of cross-instructor transferability rests on convergent qualitative pattern-matching rather than confirmed statistical equivalence. Section 5.3 identifies a controlled quantitative comparison between experimental and replication cohorts as a priority for future confirmatory work.

4.1.3. Discussion

(1). Theoretical Contributions of IDALE

IDALE extends existing educational technology and mathematics education scholarship in three theoretically significant directions.

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ISSN: 2776-7930 (Print) / 2807-3010 (Online)

First, the framework provides a design-level operationalisation of interactivity in mathematics education that moves beyond binary interactive/non-interactive distinctions. By simultaneously activating system, service, and content interactivity (Uppal & Ali, 2022) across all six principles, IDALE establishes a more precise and actionable conception of interactive mathematics learning - one specifying not merely that interaction should occur but how each dimension should be designed. This complements recent single-topic design efforts such as Acikyildiz and Kosa's (2021a) design principles for a vector-space learning environment, extending topic-level design principles into a comprehensive, empirically validated six-principle framework spanning an entire semester-length course.

Second, the integration of Tall's (2008) formal-axiomatic framework with Jonassen's (1995) digital constructivism within a single instructional design (P4 + P2) represents a theoretically novel synthesis. Previous research has treated procedural-structural development and digital exploration as largely parallel streams; IDALE demonstrates they can be mutually reinforcing when design deliberately sequences them: students explore procedures digitally, and observed structural consequences become the empirical basis for axiomatic proof construction.

Third, the five-phase learning trajectory provides a temporal architecture for deep learning in university mathematics absent from most existing frameworks. The progression from mindful procedural awareness through joyful exploration and collaborative diversification to meaningful peak interactivity and generalisation is an empirically derived sequencing logic - grounded in observed classroom data rather than a priori theory alone - contributing to the learning trajectory design literature (Simon, 1995; Clements & Sarama, 2004).

Addressing RQ4 directly, a reasonable question is why IDALE relies on general-purpose spreadsheet and video-conferencing tools (Google Sheets, Google Meet) rather than specialised dynamic mathematics platforms such as GeoGebra, MATLAB, or Maple, which offer more powerful purpose-built symbolic and graphical capabilities for linear algebra (Solano-Macías, 2023; Engelbrecht & Borba, 2024). The choice was a deliberate design trade-off rather than an oversight. GeoGebra and computer algebra systems provide superior visualisation of eigenspaces and transformations, but typically require dedicated installation, licensing, or a steeper syntax-learning curve that can itself become an extraneous cognitive load for pre-service teachers who are simultaneously learning the underlying mathematics (cf. Ishartono et al., 2022, on GeoGebra-supported self-regulated learning, where platform familiarity was a precondition for benefit rather than a given). Google Sheets, in contrast, was already familiar to all participants, is free and browser-based with no installation barrier - a material consideration in a public-university, mixed-resource Indonesian context - and natively supports the real-time multi-user collaborative editing that Principle P3 (Contextual and Task-Based Collaboration) required; comparable collaborative use of spreadsheets in mathematics instruction has been reported elsewhere (Tolboom, 2023; Tang & La, 2024). This affordance-matching argument does not establish that Google Sheets is pedagogically superior to GeoGebra or a CAS; it explains the fit between tool and context. We agree with the reviewer that a direct comparative study - IDALE implemented via Google Sheets versus an equivalent design instantiated in GeoGebra or MATLAB - would meaningfully test whether the six IDALE principles, rather than the specific software, are the active ingredient, and we identify this as a priority direction for future research (Section 5.3).

(2). Explaining IDALE's Effectiveness

The 97.2% pass rate combined with the reduction in standard deviation from 13.8 to 11.5 suggests IDALE operates through dual mechanisms. First, adaptive and differentiation provisions (P6, Characteristics 3-5) enabled lower-achieving students to accelerate across the semester while high achievers maintained performance - the equalisation effect characteristic of well-designed self-regulated learning environments (Zimmerman, 2002; Pintrich, 2004). Second, the P5 instant feedback mechanism enabled rapid error detection and correction that conventional assignment-return cycles cannot provide. As above, this interpretation is

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https://doi.org/10.35877/soshum5142

ISSN: 2776-7930 (Print) / 2807-3010 (Online)

descriptive; it is offered as a plausible mechanism consistent with the observed pattern rather than as a causally confirmed effect, pending inferential analysis of individual-level score data.

The engagement trajectory from passive in S1 to consistently collaborative and argumentative in S9-S14 reflects sociomathematical norm formation (Yackel & Cobb, 1996). IDALE's iterative collaborative and reflective provisions installed a classroom culture in which substantive intellectual exchange became the expected standard of participation. Once established around S5, these norms were self-sustaining: students who initially required explicit prompting to argue mathematically later initiated such discourse spontaneously.

The 97.2% positive environment response reflects IDALE's balance between student autonomy (P6) and cognitive scaffolding (P4, P5) - an instantiation of 'guided discovery' (Kirschner et al., 2006; Loibl & Leuders, 2018): exploratory but not minimally guided. Students experienced both the intrinsic rewards of independent discovery and the security of structural scaffolding preventing productive exploration from degenerating into confusion.

(3). Limitations

Several methodological limitations, raised directly by peer review, warrant explicit acknowledgement. First, the quantitative evidence reported here is descriptive rather than inferential: no paired-samples t-test, ANOVA, or effect-size calculation (e.g., Cohen's d) was conducted to test whether the UTS-to-UAS gain, or the difference between the experimental and replication classes, is statistically significant rather than attributable to normal variation; no independent-samples t-test or Mann-Whitney U test was used to formally compare the two cohorts. Second, internal consistency reliability (e.g., Cronbach's alpha) was not computed for the 16-item OTL or 13-item IDALE questionnaires, which were validated through expert-panel content validity (CVR $\geq$ 0.75) rather than large-sample psychometric scaling. Third, the session-by-session engagement bands in Table 6 were derived from field-note-based observational coding without a documented interval-recording protocol or a second independent coder, so no inter-rater reliability statistic (e.g., Cohen's kappa) is available. These are treated here as boundaries on the strength of the causal and comparative claims that can currently be made, not as claims that have been statistically confirmed; a fully powered confirmatory study - with individual-level paired scores, item-level questionnaire responses, and dual-coded engagement observations - is identified as the necessary next step and is now planned as a direct follow-up to this design-based cycle.

Generalisability is further bounded by sample characteristics. Participants were pre-service mathematics teachers at a single Indonesian university - a purposive, self-selected, and mathematically motivated population already favourably disposed toward digital tools (cf. Marbán & Sintema, 2021, on pre-service teachers' generally positive technology attitudes) - rather than a random sample, and no randomised or non-equivalent control group taught through conventional instruction was included for comparison. The very high pass rate (97.2%) and positive response rate (97.2%) should therefore be interpreted cautiously: they are susceptible to selection bias, social desirability bias in self-report questionnaires, and a Hawthorne or novelty effect associated with a newly introduced, closely observed instructional design, a pattern documented elsewhere in technology-based mathematics interventions (Tamur et al., 2021; Vondrová, 2024). We have not established how IDALE would perform with non-mathematics majors, engineering students, or developmental/remedial mathematics learners, who typically face the greatest difficulty with Linear Algebra and for whom motivation and prior preparation differ substantially from this sample; this is an important boundary on the transferability claims made in this paper and is flagged as a priority for future replication.

Finally, IDALE was not benchmarked against alternative dynamic mathematics platforms (e.g., GeoGebra, MATLAB, Maple) or against a conventional lecture-based control condition using the same instruments; Section 5.1 explains the contextual rationale for the Google Sheets/Google Meet implementation, but the relative contribution of the six IDALE principles versus the specific software used remains an open empirical question. Similarly,

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https://doi.org/10.35877/soshum5142

ISSN: 2776-7930 (Print) / 2807-3010 (Online)

the fifth learning-trajectory phase is described here in terms of the bounded, course-level self-regulation gains actually observed rather than as evidence of lifelong learning outcomes, which a single 14-session course cannot establish; Table 3 has been revised accordingly.

These are limitations to be addressed by future confirmatory work, not defects that undermine the present contribution. This study was designed and reported as a single, complete cycle of Design-Based Research (DBR), a methodology whose explicit purpose is to generate theoretically grounded, empirically refined design principles through iterative enactment in authentic settings, rather than to establish causal efficacy against a control condition (Design-Based Research Collective, 2003; McKenney & Reeves, 2019). Randomised or controlled comparison belongs to a later, confirmatory phase of a DBR programme, undertaken only once the design principles themselves have stabilised through cycles such as this one. Judged against that standard, the appropriate output of the present cycle is the six-principle framework and the five-phase trajectory it yields, together with descriptive evidence that the framework is implementable and associated with the outcomes reported here - not a causal claim requiring a control group. The confirmatory comparison against a conventional-instruction control group and against a GeoGebra/MATLAB implementation, named in Section 6, is the methodologically appropriate next step, not a gap that invalidates this first cycle.

5. Conclusion

This paper has presented IDALE, a six-principle instructional design framework for university-level Linear Algebra validated through a complete Design-Based Research cycle. IDALE specifies how digital tools, collaborative structures, instructional sequencing, and assessment practices should be organised to simultaneously maximise conceptual mathematical development and student engagement in a course widely recognised as a critical - and frequently difficult - transition point in undergraduate mathematics.

Empirical evidence gathered across the DBR cycle - a 97.2% pass rate, a 97.2% cumulative positive environment response, and consistent engagement of at least two-thirds of students from Session 5 onward - provides descriptive, practically meaningful support for the framework's effectiveness under authentic classroom conditions, though these findings are correlational and descriptive rather than causally confirmed through inferential statistical testing (see Section 5.3). Replication-class findings extend this evidence qualitatively to a second instructor and cohort, strengthening confidence in transferability while stopping short of formal statistical confirmation of cross-class equivalence. IDALE's principal contribution lies in transforming abstract prescriptions for 'interactive and dynamic' mathematics learning into six specific, theoretically grounded, and practically actionable design principles whose claims are intentionally scoped to the bounded, single-institution, single-semester context in which they were derived.

Future research directions include: (1) a confirmatory quantitative study using individual-level paired scores, item-level questionnaire data, and dual-coded observational data to compute inferential statistics (paired- and independent-samples tests, effect sizes, Cronbach's alpha, Cohen's kappa) that the present descriptive cycle could not support; (2) a controlled comparison of IDALE implemented via Google Sheets against equivalent designs using GeoGebra, MATLAB, or Maple, to disentangle the contribution of the six design principles from the specific software platform; (3) extending IDALE validation to non-mathematics-majors, engineering students, and developmental mathematics learners, and to other advanced mathematics courses; (4) investigating long-term impacts on mathematical identity and knowledge retention beyond the bounded, course-level self-regulation gains reported here; and (5) examining scalability in large-enrolment contexts and developing AI-enhanced adaptive extensions of IDALE's feedback and pacing mechanisms.

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